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Professor Ransom gives a simple formula which may be used as a check in connection with the ambiguous case in the solution of plane triangles. Of course it applies only to problems which yield two solutions. Apparently there is a similar formula for the analogous case of spherical triangles:

$$\tan \frac{1}{2}(c' + c'') = \tan b \cos A,$$

and another for the dual case of ambiguity.

Professor Dick discusses a relationship between the dimensions of the "King's Chamber" in the Great Pyramid, the regular pentagon, and the regular icosahedron and dodecahedron. The implication of a connection between this relationship, the occurrence of the numbers of Fibonacci in plant life, and certain mystic statements of the ancient mathematicians will perhaps seem to most readers to be somewhat far-fetched. It should also be stated that the relation between a regular pentagon and the inscription of a square in a semicircle is clearly implied by the usual construction.

I. RESOLUTION OF A CERTAIN QUINTIC EQUATION AND A GEOMETRICAL CONSTRUCTION FOR ITS ROOTS.

By C. B. HALDEMAN, Ross, Butler County, Ohio.

1. The transformation

$$y = x - \frac{a}{x}$$

will reduce the equation $y^5 + 5ay^3 + 5a^2y + 2b = 0$ to $x^{10} + 2bx^5 - a^5 = 0$; from which we find x , and consequently

$$y = \sqrt[5]{-b + \sqrt{b^2 + a^5}} + \sqrt[5]{-b - \sqrt{b^2 + a^5}}.$$

If $b^2 + a^5$ be negative it appears the roots of the given equation are trigonometrical functions of the coefficients; for the transformation

$$y = -2S\sqrt{-a}$$

will reduce the given equation to

$$16S^5 - 20S^3 + 5S = \frac{b}{a^2\sqrt{-a}};$$

and since

$$16 \sin^5 A - 20 \sin^3 A + 5 \sin A = \sin 5A,$$

we may take

$$S = \sin A, \quad \frac{b}{a^2\sqrt{-a}} = \sin 5A$$

and get

$$y = -2\sqrt{-a} \sin \frac{1}{5} \sin^{-1} \frac{b}{a^2\sqrt{-a}},$$

$$y = -2\sqrt{-a} \sin \frac{1}{5} \left(2\pi + \sin^{-1} \frac{b}{a^2 \sqrt{-a}} \right),$$

$$y = -2\sqrt{-a} \sin \frac{1}{5} \left(\pi - \sin^{-1} \frac{b}{a^2 \sqrt{-a}} \right),$$

$$y = 2\sqrt{-a} \sin \frac{1}{5} \left(\pi + \sin^{-1} \frac{b}{a^2 \sqrt{-a}} \right),$$

$$y = 2\sqrt{-a} \sin \frac{1}{5} \left(2\pi - \sin^{-1} \frac{b}{a^2 \sqrt{-a}} \right).$$

2. The equation

$$xy\sqrt{-(b^2 + a^5)} = a^2y^3 + by^2 + 3a^3y + 2ab \quad (1)$$

represents a real trident of Newton¹ when $b^2 + a^5$ is negative, and

$$x^2 + y^2 = -4a \quad (2)$$

is the equation of a circle, which will be real when a is negative. Eliminating x from (1) by means of (2), the result may be placed under the form

$$(y^5 + 5ay^3 + 5a^2y + 2b)(a^2y + 2b) = 0.$$

The first of these factors is identical with the quintic expression given above and has five real roots when $b^2 + a^5$ is negative. From this it appears the five real roots may be represented by the ordinates of the intersections of a trident and a circle. The five intersections, whose ordinates are the five real roots of this equation, are the vertices of a regular pentagon inscribed in a circle whose radius is $2\sqrt{-a}$, as may be seen by reference to the above values of y .

II. CERTAIN MATHEMATICAL FEATURES OF THERMODYNAMICS.

By J. E. TREVOR, Cornell University.

Let e , v , s denote the energy, volume, and entropy of unit mass of a body of homogeneous fluid in a state of thermodynamic equilibrium under the pressure p at the temperature θ . The energy e is then a continuous function of v , s , and equilibrium subsists when and only when

$$(1) \quad de = -pdv + \theta ds.$$

Hence the conditions of equilibrium are the equations

$$(2) \quad -p = \partial e / \partial v, \quad \theta = \partial e / \partial s.$$

The state of equilibrium is stable, with respect to small displacements, when

¹ One of the four canonical forms (no. 108) to which Newton has reduced the general equation of cubic curves in his *Enumeratio linearum tertii ordinis*, first printed in Newton's *Optics*, London, 1704.—EDITOR.